Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department **Qualifying Courses (Mathematics)**		Final Term Exam Date: January 2, 2018 Course: Linear Algebra, EMM 402 Duration: **3** hours
• Answer **All** questions The exam consists of one page		• No. of questions: 5 Total Mark: 60

Question 1

(a)State the definitions : Singular matrix, Symmetric matrix, Trace of matrix. 6

(b)Prove that: If A is a square matrix. Then A and A\` have the same eigenvalues. 3

(c)Show that the eigenvalues of the matrix : $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ are real numbers, 6

where a, b, c are real numbers.

Question 2

(a)Determine the linearly independent and linearly dependent: 3

(i) u = (2, 2), v = (1, 3) (ii) u = (1, 0, 3), v = (1, 2, 2), w = (0, 2, – 1)

(b) If $A = \begin{bmatrix} 2 & 4 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

Find, if possible, A + B, A.B, A.A , $A^t.A$, $|A^t.A|$. 7

Question 3

Write the following expressions in matrix form and determine the type : 9

(a) $P = 5x^2 + 2y^2 + 3z^2 + 2xy - 2xz + yz$

(b) $P = xy + 2xz - 2yz - 4x^2 - 3y^2 - 2z^2$

(c) $P = xy - 6xz + 2yz + x^2 + 2y^2 + 2z^2$

Question 4

(a)Write the Hessian matrix of : $f(x, y, z) = y\,e^{2x} + x\cosh y + x^3 \sin z$. 3

(c) If $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$. Find the eigenvalues and the eigenvectors of A and find 12

$f(A) = A^n$. Also, find A^{-1} from Hamilton's equation.

Question 5

(a)Write the equations: $a_{11}x + a_{12}y + a_{13}z = b_1$, $a_{21}x + a_{22}y + a_{23}z = b_2$,

$a_{31}x + a_{32}y + a_{33}z = b_3$ in matrix form and state the types of solutions. 3

(b)Solve the linear system :

2x – y + 2z = 1, x + y – 3z = 0, x – y + z = – 6, 3x – z = 1. 4

(c)Write the matrix of the L.T. $A: R^3 \to R^2$ where $AX = \begin{bmatrix} 2x - y + z \\ x + y - z \end{bmatrix}$ 4

and find its kernel.

Good Luck *Dr. Mohamed Eid*

Model Answer

<u>Answer of Question 1</u>

(a) A square matrix A is singular if |A| = 0

A square matrix A is symmetric if A` = A

The trace of a A square matrix is the sum of elements of its diagonal.

---*6-Marks*

(b) Since $(A + B)` = A` + B`$  and  $|A| = |A`|$

Then $|(A - \lambda I)`| = |A` - \lambda I| = |A - \lambda I|$

Then A and A` have the same eigenvalues.

---*3-Marks*

(c)Proof

---*6-Marks*

<u>Answer of Question 2</u>

(a)(i) au + bv = a(2, 2) + b(1, 3) = (2a + b, 2a + 3b) = 0. Then a = b = 0.

Then u and v are linearly independent.

(ii) au + bv + cw = a(1, 0, 3) + b(1, 2, 2) + c(0, 2, –1)

= (a + b , 2b+ 2c, 3a + 2b – c) = 0

Then a = – b = c . Then u, v and w are linearly dependent.

---*3-Marks*

(b) $A.B$, $A.A$ are not exist.

$$A + B = \begin{bmatrix} 3 & 4 & 4 \\ 5 & 2 & 4 \end{bmatrix}, \quad A`.A = \begin{bmatrix} 8 & 4 & 8 \\ 4 & 0 & 12 \\ 10 & 6 & 6 \end{bmatrix}, \quad |A`.A| = 0$$

---*7-Marks*

<u>Answer of Question 3</u>

(a) $P = X`AX = [x \quad y \quad z]\begin{bmatrix} 5 & 1 & -1 \\ 1 & 2 & \frac{1}{2} \\ -1 & \frac{1}{2} & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is positive definite.

(b) $P = X`AX = [x \quad y \quad z]\begin{bmatrix} -4 & 1/2 & 1 \\ 1/2 & -3 & -1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is negative definite.

(C) $P = X`AX = [x \quad y \quad z]\begin{bmatrix} 1 & 1/2 & -3 \\ 1/2 & 2 & 1 \\ -3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is indefinite.

---*9-Marks*

<u>Answer of Question 4</u>

(a) $H = \begin{bmatrix} 4y\, e^{2x} + 6x \sin z & 4e^{2x} & 3x^2 \cos z \\ 4e^{2x} & x\, \cosh y & 0 \\ 3x^2 \cos z & 0 & -x^3 \cos z \end{bmatrix}$

---*3-Marks*

(b) $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 4 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) - 4 = \lambda^2 - \lambda - 6 = 0$

Then, the eigenvalues are: $\lambda_1 = 3, \quad \lambda_2 = -2.$

From the equation, $\begin{bmatrix} 2-\lambda & 1 \\ 4 & -1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For $\lambda_1 = 3$, $\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $-x + y = 0, \quad 4x - 4y = 0$

Then $x = y =$ any number except 0

Put $y = 1$, we get $x = 1$ and the eigenvector is : $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $\lambda_2 = -2$, $\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $4x + y = 0, \quad 4x + y = 0$

Then $y = -4x =$ any number except 0

Put $x = 1$, we get $y = -4$ and the eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

Then $T = \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$ and $T^{-1} = -\frac{1}{5}\begin{bmatrix} -4 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$

Then $A^n = T\begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix} T^{-1} = \begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix} . \frac{1}{5}\begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$

$$= \frac{1}{5}\begin{bmatrix} 4(3)^n + (-2)^n & (3)^n - (-2)^n \\ 4(3)^n - 4(-2)^n & (3)^n + 4(-2)^n \end{bmatrix}$$

The Hamilton's equation : $A^2 - A - 6I = 0$. Then $A^{-1} = \frac{1}{6}\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$

--*12-Marks*

<u>Answer of Question 5</u>

(a)The matrix form AX = B : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

If $G = [A \ \vdots \ B]$

This system has three types of solutions :

(i)One solution if rank A = rank G = 3.

(ii)Infinite number of solutions if rank A = rank G < 3.

(iii)No solution if rank A ≠ rank G.

--*3-Marks*

(b) $G = \left[\begin{array}{ccc:c} 1 & 1 & -3 & 0 \\ 1 & -1 & 1 & -6 \\ 2 & -1 & 2 & 1 \\ 3 & 0 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

It has no solution because rank A = 2 and rank G = 3.

--*4-Marks*

(c)The matrix of transformation is : $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$$\text{Ker A} = \left\{ X \in R^3 : \ X = \begin{bmatrix} 0 \\ y \\ y \end{bmatrix}, y \in R \right\}$$

--*4-Marks*

Dr. Mohamed Eid